# PRAGATHI...THE SCHOOL

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| ***Date : 18/10/2023*** | ***Mid-Term*** | ***Time allowed : 3Hr 15Min*** |
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| ***MATHEMATICS(Standard) - 041*** | | ***Maximum Marks: 80*** |

## Class:-XII

**General Instructions:**

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has **18 MCQ's** and **02** Assertion-Reason based questions of **1** mark each.
3. Section B has **5** Very Short Answer (VSA)-type questions of **2** marks each.
4. Section C has **6** Short Answer (SA)-type questions of **3** marks each.
5. Section D has **4** Long Answer (LA)-type questions of **5** marks each.
6. Section E has **3** source based/case based/passage based/integrated units of assessment of **4** marks each with sub-parts

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### Section –A
### (Multiple Choice Questions)
### Each question carries 1 mark

1. **The relation R in the set of real numbers defined as R = {(a, b) ∈ R × R : 1 + ab > 0} is**
   (a) reflexive and transitive (b) symmetric and transitive
   (c) reflexive and symmetric (d) equivalence relation
2. **Let the function 'f ' be defined by $f(x) = 5x^2 + 2$, ∀ x ∈ R. Then 'f ' is**
   (a)onto function (b) one-one, onto function
   (c) one-one, into function (d) many-one, into function
3. **Let set X = {1, 2, 3} and a relation R is defined in X as : R = {(1, 3), (2, 2), (3, 2)}, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are**
   (a){(1, 1), (2, 3), (1, 2)} (b) {(3, 3), (3, 1), (1, 2)}
   (c) {(1, 1), (3, 3), (3, 1), (2, 3)} (d) {(1, 1), (3, 3), (3, 1), (1, 2)}
4. **A is a skew-symmetric matrix and a matrix B such that B'AB is defined, then B'AB is a:**
   (a)symmetric matrix (b) skew-symmetric matrix
   (c) Diagonal matrix (d) upper triangular symmetric
5. **If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A$ is**
   (a)I (b) 2A (c) 3I (d) A
6. **If A and B are square matrices of order 3 such that |A| = 1 and |B| = 3, then the value of |3AB| is:**
   (a)3 (b) 9 (c) 27 (d) 81
7. **Let A be a non-singular matrix of order (3 × 3). Then |adj.A| is equal to**
   (a)|A| (b) $|A|^2$ (c) $|A|^3$ (d) 3|A|

*8.* **A and B are invertible matrices of the same order such that $|(AB)^{-1}| = 8$, If $|A| = 2$, then $|B|$ is**

(a) 16 (b) 4 (c) 6 (d) 1/16

*9.* **A function f is said to be continuous for $x \in R$, if**

(a) it is continuous at x = 0 (b) differentiable at x = 0

(c) continuous at two points (d) differentiable for $x \in R$

*10.* **The domain of the function defined by $\sin^{-1}\sqrt{x-1}$ is**

(a) [1, 2] (b) [–1, 1] (c) [0, 1] (d) none of these

*11.* **The value of $\tan^2\left(\sec^{-1}2\right)+\cot^2\left(\operatorname{cosec}^{-1}3\right)$ is**

(a) 5 (b) 11 (c) 13 (d) 15

*12.* **Derivative of sin x with respect to log x, is**

(a) $\frac{x}{\cos x}$ (b) $\frac{\cos x}{x}$ (c) $x.\cos x$ (d) $x^2.\cos x$

*13.* **The function 'f ' defined by** $f(x)=\begin{cases}\frac{x^3-8}{x-2}, & x\neq 2\\ 12, & x=2\end{cases}$ **is**

(a) not continuous at x = 2 (b) continuous at x = 2

(c) not continuous at x = 3 (d) not continuous at x = – 2

*14.* **If $3\tan^{-1}x+\cot^{-1}x=\pi$, then x equals**

(a) 0 (b) 1 (c) –1 (d) 1/2

*15.* **A function** $f(x)=\begin{cases}\frac{\sin x}{x}+\cos x, & x\neq 0\\ 2k, & x=0\end{cases}$ **, is continuous at x = 0 for**

(a) k=1 (b) k=2 (c) k=1/2 (d) k=3/2

*16.* *If* $\begin{vmatrix}4 & 1\\ 2 & 1\end{vmatrix}^2=\begin{vmatrix}3 & 2\\ 1 & x\end{vmatrix}-\begin{vmatrix}x & 3\\ -2 & 1\end{vmatrix}$, **then x equals**

(a) 6 (b) 3 (c) 7 (d) 1

*17.* **Let A = {a, b, c}, then the total number of distinct relations in set A are**

(a) 64 (b) 32 (c) 256 (d) 512

*18.* **The diagonal elements of a skew symmetric matrix are**

(a) all zeroes (b) are all equal to some scalar k($\neq$ 0)

(c) can be any number (d) none of these

**ASSERTION-REASON BASED QUESTIONS**

**In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).**

**Choose the correct answer out of the following choices.**

**(a) Both (A) and (R) are true and (R) is the correct explanation of (A).**

**(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).**

**(c) (A) is true but (R) is false.**

**(d) (A) is false but (R) is true**

*19.* **Assertion (A):** f(x)= |x – 3| is continuous at x = 0.

**Reason (R):** f(x) = |x – 3| is differentiable at x = 0.

*20.* **Assertion (A):** The value of determinant of a matrix and the value of determinant of its transpose are equal.

**Reason (R):** The value of determinant remains unchanged if its rows and columns are interchanged

**Section –B**

**[This section comprises of very short answer type questions (VSA) of 2 marks each]**

*21.* Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$

**OR**

Find the domain of $\sin^{-1}\left(x^2-4\right)$

*22.* Prove that the Greatest Integer Function $f: R \to R$, given by $f(x) = [x]$ is neither one-one nor onto. Where **[x]** denotes the greatest integer less than or equal to x.

*23.* Find X and Y, if $X+Y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$ & $X-Y=\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$

*24.* Area of a triangle with vertices **(k, 0), (1, 1) and (0, 3**) is **5** sq units. Find the value(s) of k.

**OR**

Find the value of x, such that the points (**0, 2), (1, x) and (3, 1)** are collinear

*25.* Differentiate **5sin x**, with respect to x.

**Section – C**

**[This section comprises of short answer type questions (SA) of 3 marks each]**

*26.* Show that the relation S in the set R of real numbers, defined as $S = \{(a, b) : a, b \in R \text{ and } a \le b^3\}$ is neither reflexive, nor symmetric, nor transitive

**OR**

Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T1, T2) : T1 \cong T2\}$. Show that R is an equivalence relation.

*27.* Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

*28.* If $A=\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

**OR**

If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

***29.*** Show that $\mathbf{A}=\begin{bmatrix}2 & -3\\ 3 & 4\end{bmatrix}$ satisfies the equation $\mathbf{x^2-6x+17=O}$. Hence find $\mathbf{A^{-1}}$

***30.*** Show that the function $f:R\to R$ defined by $f(x)=\dfrac{x}{x^2+1},\forall x\in R$ is neither one-one nor onto.

***31.*** Show that the function **f(x) = |x – 3|, x ∈ R** is continuous but not differentiable at x = 3

**OR**

Differentiate $\mathbf{x^{x.\cos x}+\dfrac{x^2+1}{x^2-1}}$ . w.r.t.x

**Section –D**

**[This section comprises of long answer type questions (LA) of 5 marks each]**

***32.*** Let **N** be the set of all natural numbers and $R$ be a relation on $\mathbf{N\times N}$ defined by $(a,b)R(c,d)\Leftrightarrow ad=bc$ for all $(a,b),(c,d)\in N\times N$ . Show that R is an equivalence relation on $\mathbf{N\times N}$. Also, find the equivalence class of $(2,6)$ , i.e., $\left[(2,6)\right]$

**OR**

Show that the function $f:R\to\{x\in R:-1<x<1\}$ defined by $f(x)=\dfrac{x}{1+|x|},x\in R$, is one-one and onto function.

***33.*** Using the matrix method, solve the following system of linear equations :

$\mathbf{\dfrac{2}{x}+\dfrac{3}{y}+\dfrac{10}{z}=4,\ \dfrac{4}{x}-\dfrac{6}{y}+\dfrac{5}{z}=1,\ \dfrac{6}{x}+\dfrac{9}{y}-\dfrac{20}{z}=2}$

***34.*** If $\mathbf{f(x)}=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, \text{ if } x<\dfrac{\pi}{2}\\ a, \text{ if } x=\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2}, \text{if } x>\dfrac{\pi}{2}\end{cases}$ If f(x) be a continuous function at **x =** $\dfrac{\pi}{2}$, find a and b.

***35.*** Given $\mathbf{A}=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\ \&\ \mathbf{B}=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$, verify that **BA = 6I**, how can we use the result to find the values of x, y, z from given equations **x – y = 3, 2x + 3y + 4z = 17, y + 2z = 17**

**OR**

If $\mathbf{A}=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find $A^{-1}$ and hence solve the system of linear equations:

**2x – 3y + 5z = 11, 3x + 2y – 4z = –5; x + y – 2z = –3**

## Section –E

**[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts.**

**The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively.**

**The third case study question has two sub parts of 2 marks each.)**

***36.*** Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted that possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes.

A = {S, D}, B = {1, 2, 3, 4, 5, 6}

(i) Let R : B $\rightarrow$ B be defined by **R = {(x, y) : y is divisible by x}.** Show that relation R is reflexive and transitive but not symmetric.

(ii) Let R be a relation on B defined by

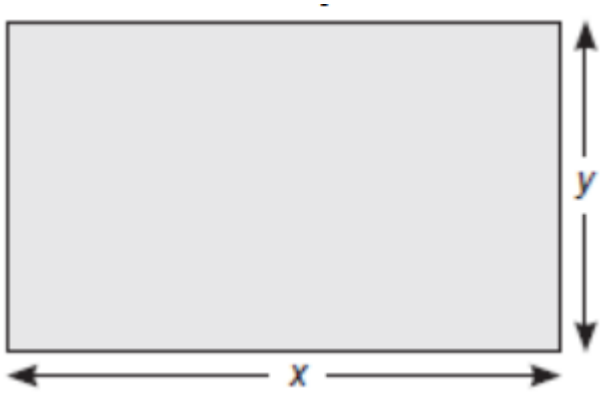
**R = {(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)}.** Then check whether R is an equivalence relation.

(iii) Raji wants to know the number of functions from **A to B**. How many number of functions are possible?

**OR**

(iii) Raji wants to know the number of relations possible from **A to B**. How many numbers of relations are possible

***37.*** Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 $m^2$ .




Based on the information given above, answer the following questions:

(i) Find the equations in terms of x and y (1)

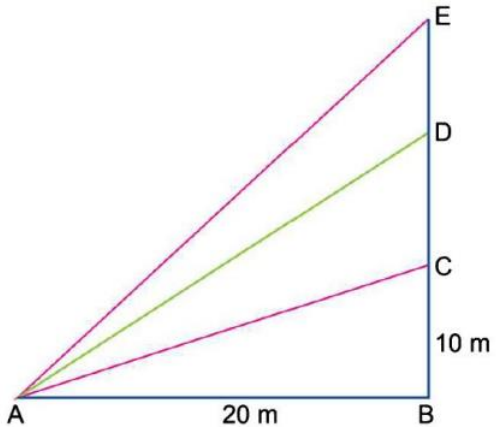
(ii) Find the value of x (length of rectangular field). (1)

(iii) Find the value of y (breadth of rectangular field).

**OR**

(iii)How much is the area of rectangular field?

***38.*** The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "A" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. "C" is at the height of 10 metres from the ground level. For the viewer A, the angle of elevation of "D" is double the angle of elevation of "C" The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer. Look at the figure given and based on the above information answer the following:




Based on the above information, answer the following questions:

(i) Find the measure of ∠DAB

(ii) Find the measure of ∠EAB